THE INFLUENCE OF INERTIAL INSTABILITY UPON TRANSOSONDE TRAJECTORIES AND SOME FORECAST IMPLICATIONS

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ABSTRACT

Trajectory configurations resulting from the passage of transosondes through regions of inertial (dynamic) instability are considered. It is shown that cusped trajectories and trajectories with small anticyclonic loops occur downstream from such regions, with the particular trajectory configuration most likely dependent upon the magnitude of the pressure gradient in the area where the transosonde velocity approaches zero. The large geostrophic departures existing in, and downstream from, regions of inertial instability make conventional trajectory estimations within such regions difficult, if not impossible. Cases are discussed wherein there is evidence that large ageostrophic flows associated with regions of inertial instability are precursors of changes in pressure pattern.

1. INTRODUCTION

Evidence for inertial oscillations along transosonde trajectories has been presented in a previous paper [1]. In this paper emphasis is placed upon transosonde trajectories which pass through regions of inertial instability. Within such regions the inertial period is infinite, and since there is no obvious force acting to restore the flow to geostrophic or gradient equilibrium, large ageostrophic or agradient flows would be anticipated.

If the basic flow is geostrophic, van Mieghem [2] and others have shown that inertial or dynamic instability exists when the absolute (geostrophic) vorticity computed on an isentropic surface is negative. If the basic flow is curved, the criterion for inertial instability is known only for the case of a steady symmetrical vortex. Under such conditions, inertial instability exists if, on an isentropic surface, the absolute vorticity is negative and the flow is normal, or the absolute vorticity is positive and the flow is abnormal; that is, possesses anticyclonic rotation in space [3]. A derivation of the latter criterion has recently been presented by Alaka [4] in a comprehensive article dealing with abnormal or anomalous winds, and will not be repeated here. However, it is important to note that negative absolute vorticity is usually attained through the existence of large anticyclonic wind shears, whereas the criterion for abnormal flow is independent of horizontal wind shear.

The purpose of the paper is to illustrate, through the use of transosonde trajectories, the form and magnitude of the ageostrophic flows resulting from inertial instability, and to point out some possible forecast implications thereof. Throughout this discussion it must be remembered that the transosondes did not partake of the vertical air motions, and consequently the horizontal projections of the 3-dimensional air parcel trajectories may have somewhat different configurations than the illustrated transosonde trajectories. Furthermore, while the criterion for inertial instability is prescribed for isentropic surfaces, we are assuming it can be applied to isobaric surfaces with little approximation.

2. INERTIAL INSTABILITY AND TRAJECTORY CUSPS

In the transosonde program, the first clear-cut example of transosonde passage through a region of inertial instability occurred on flight 990, launched from Minneapolis in April 1953 for flight at 300 mb. Figure 1 shows a portion of this flight trajectory with smoothed positions, and the winds derived therefrom, indicated at 4-hour intervals. These smoothed positions (and all other positions utilized in this paper) were obtained by a one-two-one weighting of successive 2-hourly latitudes and longitudes obtained by Federal Communications Commission radio-direction-finding intercepts on the transosondes. It is seen from figure 1 that as flight 990 crossed the east coast of the United States it was embedded in a region of strong anticyclonic wind shear, as shown both by the contour spacing of the NAWAC analysis and the wind speed difference between Hempstead Air Force Base on Long Island (150 kt.) and Washington, D.C. (90 kt.). With such straight contours a good approximation to the geostrophic wind shear is given by the more objectively determined anticyclonic wind shear between Hempstead and Washington, which amounted to 60 kt. in 180 n.mi. or -0.90×10^{-4} sec.$^{-1}$ The earth's vorticity about the local vertical at this latitude is 0.87×10^{-4} sec.$^{-1}$ Taking into account the additional cyclonic vorticity of the flow due to zonal movement about the spherical earth and noting that for

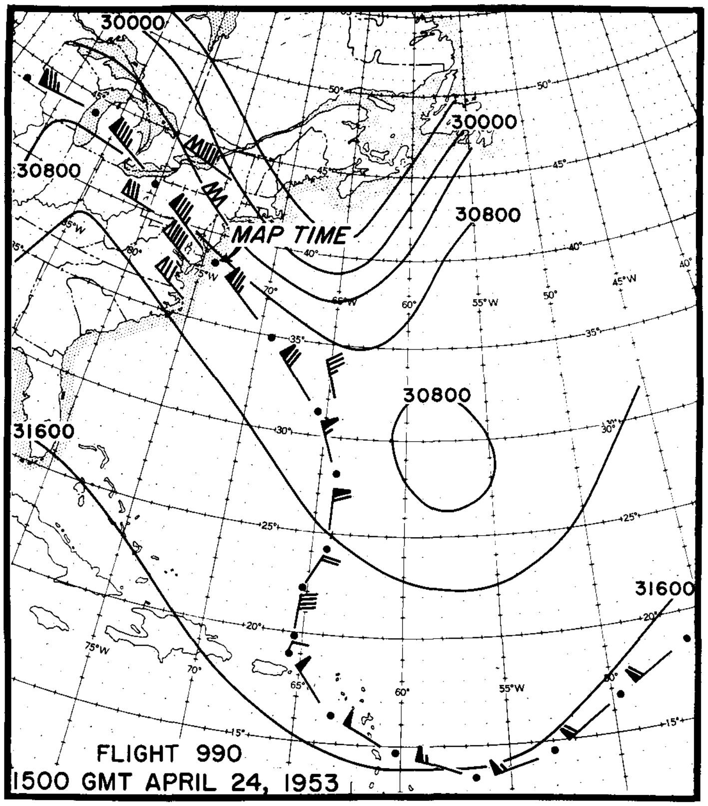

FIGURE 1.—Portion of trajectory of 300-mb. transosonde flight 990, with smoothed positions and winds (4-hour intervals) superimposed on the 300-mb. NAWAC analysis for 1500 GMT, April 24, 1953.

all practical purposes there was no curvature vorticity, we see that at the position of the transosonde at map time negative absolute vorticity existed or was closely approximated.

Assuming that negative absolute vorticity actually did exist, the question arises as to how it was attained, since horizontal divergence is incapable of producing such an extreme condition. In this case the temperature pattern (not shown in fig. 1, but see fig. 12 of reference [5]) suggests that the negative absolute vorticity could have resulted from the twisting into the vertical of vortex tubes associated with vertical wind shear. Thus, at the jet core a descending motion of about 5 cm. sec. $^{-1}$ was derived from the temperature change following a hypothetical air parcel (adiabatic method of determining vertical motion) while along the transosonde trajectory the interpolated temperatures indicate, if anything, a slight ascending motion. With the given vertical wind shear of about 10 m. sec. $^{-1}$ in 1000 meters, this leads to a vorticity change of -5×10^{-5} sec. $^{-1}$ in 6 hours, a value easily sufficient to produce the negative absolute vorticity observed. This twisting effect appears to terminate off the east coast of the United States and there the absolute vorticity at the position of the transosonde became more nearly equal to zero.

Although the sparsity of 300-mb. data in the area subsequently traversed by flight 990 makes quantitative evaluations difficult, it is apparent from figure 1 that there was a tendency for conservation of zero absolute vorticity following the transosonde with the anticyclonic shear vorticity transforming into anticyclonic curvature vorticity. Thus, at latitude 25°, the curvature vorticity suggested by the transosonde trajectory is nearly half the earth's vorticity about the local vertical so that if anticyclonic shear of one-third to one-half the magnitude found between Hempstead and Washington existed, the absolute vorticity at the position of the transosonde was still zero. Such a tendency toward conservation of zero absolute vorticity is not too surprising when it is noted, on the basis of the simplified vorticity equation, that if the absolute vorticity is zero, it is insensitive to the field of horizontal divergence.

As a consequence of this tendency toward conservation of zero absolute vorticity, the transosonde moved much farther south than would be expected from the contour pattern of figure 1, finally attaining a latitude of 13°. Conventional data are too sparse to confirm that this extension of the trajectory to southerly latitudes was reflected in a southward intensification of the associated pressure trough, although there is a suggestion along this line. What we wish to emphasize here is that just north of Puerto Rico the 4-hour average transosonde speed decreased to 10 kt. and a sort of cusp in the trajectory appears, separating the portion of the trajectory with anticyclonic curvature from the portion with cyclonic curvature. The unsmoothed transosonde positions even indicated a small anticyclonic loop in this region just north of Puerto Rico. Unfortunately, flight 990 was not released far enough upstream to sample the ridge south and west of the Great Lakes so that the magnitude of the anticyclonic angular velocity on the ridge is not known with certainty. It must have been quite large, however, judging from the contour configuration.

In summary, this early transosonde flight tends to confirm the existence of trajectory cusps or small anticyclonic loops downstream from regions of large anticyclonic wind shear and sharply curved ridges, as found by Gustafson [6] utilizing analytic flow models and an analog computer. Furthermore, flight 990 clearly illustrates the large deviation from geostrophic flow to be expected under conditions of inertial instability.

A more striking example of a trajectory cusp resulting from transosonde passage through a region (or regions) of inertial instability is illustrated in figure 2, which shows a portion of the 300-mb. trajectory of flight 36′ (one of the early transosonde flights from Japan) superimposed on 12-hourly segments of NAWAC analyses. Note the strong deceleration experienced by the transosonde over and to the south of California. The 2-hour average speed along the trajectory decreased from 130 kt. to 25 kt. in 8 hours. Such a deceleration should be associated, according to the tangential equation of motion, with a mean flow toward high pressure of about 60 kt. so that, as can be seen from the figure, the flow was directed nearly at right angles to the contours during the period of deceleration. The cusp-like form of the trajectory is apparent with the cusp separating the region of rapid transosonde deceleration from the region of rapid transosonde acceleration.

Upstream from the cusp there is evidence for inertial instability due both to the occurrence of abnormal flow in a region of positive absolute vorticity (Pacific coast ridge) and to the occurrence of negative absolute vorticity in a region of normal flow (California-Nevada area). Thus, with regard to the latter phenomenon, note that at Bishop, Calif. the wind speed was 130 kt. whereas at Merced the wind speed was 85 kt., yielding an anticyclonic wind shear of 45 kt. in 100 n. mi. or -1.20×10^{-4} sec.$^{-1}$ The earth's vorticity about the local vertical at this latitude is 0.83×10^{-4} sec.$^{-1}$ The cyclonic (geostrophic) curvature vorticity does not appear sufficient to overcome this difference so that there is evidence for the existence of negative absolute vorticity between these two stations and farther downstream as well. With regard to the Pacific coast ridge, during the 12-hour period from 2200 GMT on February 15 to 1000 GMT on February 16 the transosonde-derived wind direction changed by 150°, corresponding to an (anticyclonic) angular velocity of -0.60×10^{-4} sec. $^{-1}$. This is identical with the cyclonic angular velocity of the earth about the local vertical at this latitude, so that for 12 hours the flow was on the verge of being abnormal. Under such conditions the wind speed has twice the geostrophic value. During the 4-hour period from 0600 to 1000 GMT on February 16 the flow was slightly abnormal if the transosonde positions are believed implicitly. Under these conditions the wind speed would have more than twice the geostrophic value.

In summary, then, on flight 36′ a cusped trajectory appeared downstream from a ridge where the flow was barely abnormal, or on the verge of being abnormal, and downstream from a region where the absolute vorticity was probably negative. Consequently, it is not possible to relate the instability illustrated by the cusped trajectory to one, rather than the other, phenomenon. However, it might be noted that two days later along this same trajectory (over the North Atlantic) the transosonde traced out a more pronounced cusp, and while there is evidence that the transosonde was embedded in a region of negative absolute vorticity during the preceding period of extreme deceleration (180 kt. in 8 hours), it does not appear that the flow was abnormal on the upstream ridge in this case.

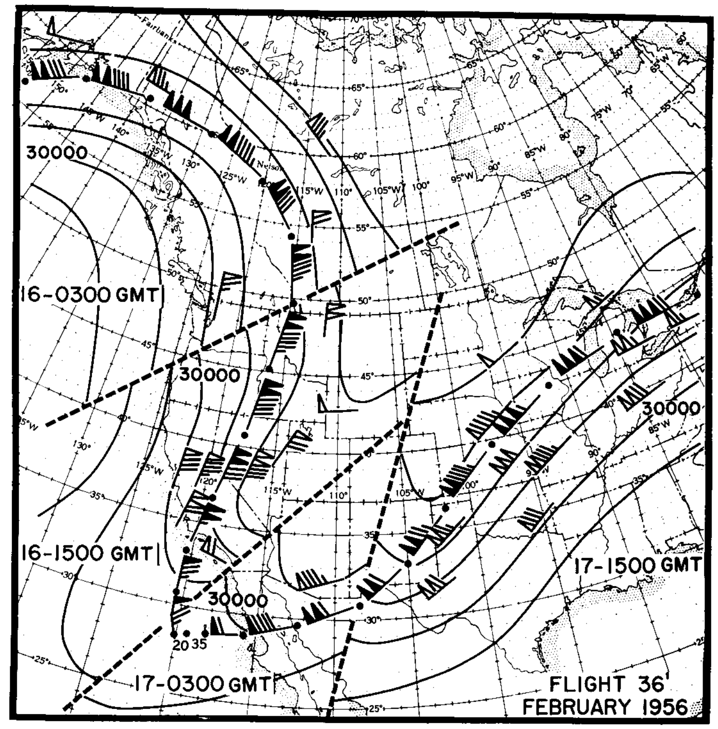

FIGURE 2.—Portion of trajectory of 300-mb. transosonde flight 36′, with smoothed positions and winds (2-hour intervals) superimposed on 12-hourly segments of 300-mb. NAWAC analyses. At the trajectory cusp the transosonde speed (in knots) is given by number.

3. INERTIAL INSTABILITY AND TRAJECTORY LOOPS

In the previous section evidence for a relationship between inertial instability and trajectory cusps was presented. There is also limited evidence that under somewhat similar conditions the transosonde trajectory may take on the form of a small anticyclonic loop, so that at some point the flow is antigeostrophic. The best example of this phenomenon is illustrated in figure 3, which shows 2-hour average winds plotted along a portion of the 250-mb. trajectory of transosonde flight 228, one of the last flights released from Japan. Especially to be noted in this figure is the trajectory configuration over Tennessee, where the transosonde traced out a portion of an anticyclonic loop before the functioning of the pre-set termination device. This undoubtedly represents the most inopportune termination in transosonde history. The transosonde was tracing out this small anticyclonic loop at the rate of about 90° of arc per 6 hours which, if continued, would have resulted in loop closure in 24 hours, quite close to the inertial period for the latitude (21 hours). In some ways this loop resembles a pure inertial circle. The radius of curvature of such a circle is given by the ratio of speed and Coriolis parameter [7], and taking the average speed during the last 12 hours of flight (12 kt.), we find the appropriate radius of curvature is 1° latitude, very close to that observed in figure 3.

On flight 228, the transosonde-derived wind direction at the pressure crest changed by 60° in 4 hours, corresponding to an (anticyclonic) angular velocity of -0.73×10^{-4} sec. $^{-1}$ This is considerably greater than the cyclonic angular velocity of the earth about the local vertical at this latitude (0.60×10^{-4} sec. $^{-1}$), so that there is stronger evidence for the existence of abnormal flow on this flight than on flight

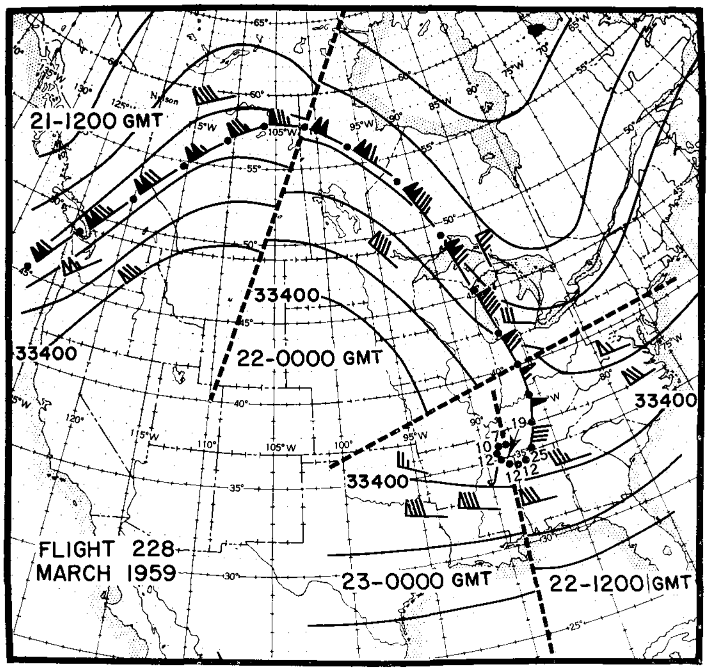

FIGURE 3.—Position of trajectory of 250-mb. transosonde flight 228, with smoothed positions and winds (2-hour intervals) superimposed on 12-hourly segments of 250-mb. NAWAC analyses. In the anticyclonic loop the transosonde speed (in knots) is given by number.

36′. However, the evidence for negative absolute vorticity on flight 228 is not as convincing as in the previous two cases. In particular, the existence of negative absolute vorticity can not be shown by winds but only by geostrophic vorticity evaluations from the pressure fields south of the Great Lakes on the two maps for March 22, 1959.

The question immediately arises as to why, in the case of flight 228, the trajectory assumed the form of a small anticyclonic loop whereas in the case of flight 36′ the trajectory assumed the form of a cusp. The difference probably lies in the magnitude of the pressure gradient in the area where the transosonde velocity became very small. Thus, on flight 228, this gradient might be considered sufficiently small that inertial flow (which, by definition, is flow in a region of zero pressure gradient) could have been approximated; while this was not true on flight 36′. An alternate, but more remote, possibility is that the looped trajectory is a consequence of the flow on the ridge on flight 228 definitely having been abnormal (wind speed on the ridge more than twice the geostrophic value) rather than more or less on the dividing line between normal and abnormal, as on flight 36′. This alternative implies that the rarity of small anticyclonic loops of the kind evidenced on flight 228 is correlated with the equal rarity of abnormal flow on large-scale ridges. There could be a tenuous connection here with the work of Kao and Neiburger [8] who have shown theoretically that if, in a pressure field of equally-spaced straight contours, the initial geostrophic deviation equals the geostrophic wind (wind speed equal to twice the geostrophic value) the trajectory assumes the form of a cycloid (cusped trajectories), whereas if the initial geostrophic deviation is greater than the geostrophic wind (wind speed more than twice the geostrophic value) the trajectory possesses small anticyclonic loops.

In passing it is of interest that some confirmation for the small anticyclonic loop on flight 228 is offered by the direction of the weak wind (2–3 kt.) in northern Mississippi. By itself this wind direction would probably be dismissed as incorrect, but the trajectory shows that basically it is correct. Its weakness relative to the transosonde-derived wind may result from its average over a 2000-foot layer in a region where large circulation variations in the vertical might be expected.

In summary of flight 228, we have hypothesized that the small anticyclonic loop in this trajectory is most likely an inertial circle, made possible by the very weak pressure gradient in the area where, subsequent to transosonde passage through a region of inertial instability, the transosonde velocity approached zero. A less likely alternative is that the loop is due to passage of the transosonde through a region of negative absolute vorticity after its passage through a ridge on which the flow was definitely abnormal. According to this latter reasoning and collating the results obtained from flights 36′ and 228, a trajectory loop would result from the feeding of a wind of more than twice geostrophic value into a region of negative absolute vorticity, whereas a cusped trajectory would result from the feeding of a wind equal to twice the geostrophic value into a region of negative absolute vorticity.

4. TRAJECTORY ESTIMATES IN REGIONS OF INERTIAL INSTABILITY

Inasmuch as large geostrophic deviations have been shown to exist in and downstream from regions of inertial instability, it is obvious that trajectory estimations in such regions are hazardous, if not impossible. In particular, any trajectory estimations based upon the geostrophic assumption are meaningless, as can be seen by reference to figure 1 and flight 990. As a striking example of the tremendous trajectory errors which may arise in regions of inertial instability, figure 4 shows the comparison between the 300-mb. trajectory of transosonde flight NL–1 (dashed line), launched from Vernalis, Calif. into such a region of instability, and the forecast trajectory (dotted line) made by the numerical computer at Suitland, Md., based on the synoptic map illustrated in the figure (the map closest to transosonde launch time). In this case the forecast trajectory was not based on the geostrophic assumption but on the balanced wind assumption, which automatically introduces difficulties in regions of large (relative) anticyclonic vorticity. For details of the trajectory forecast method, see the paper by Hubert, Wolff, and Cave [9].

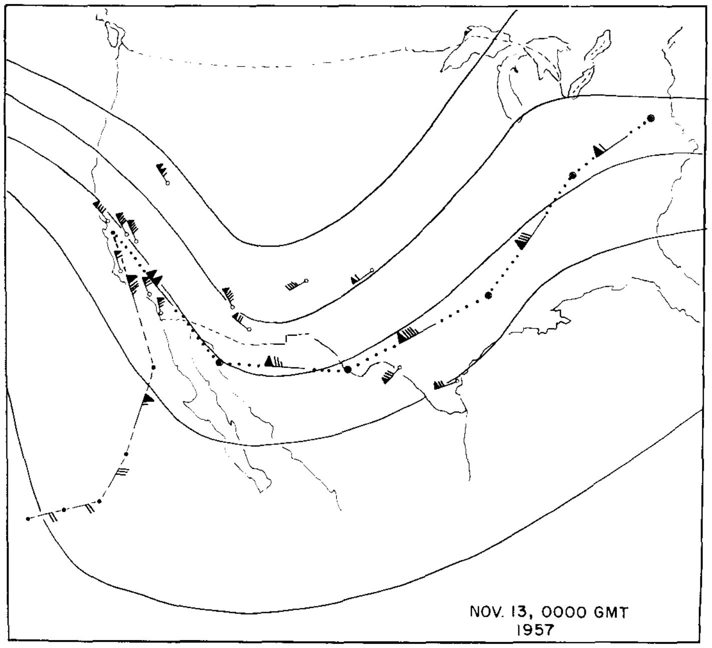

FIGURE 4.—Trajectory of 300-mb. transosonde flight NL-1 (dashed line) and numerically forecast 300-mb. trajectory (dotted line). Positions and winds for actual and forecast trajectories indicated at 6-hour intervals. The NAWAC analysis is for the synoptic map closest to transosonde launch time.

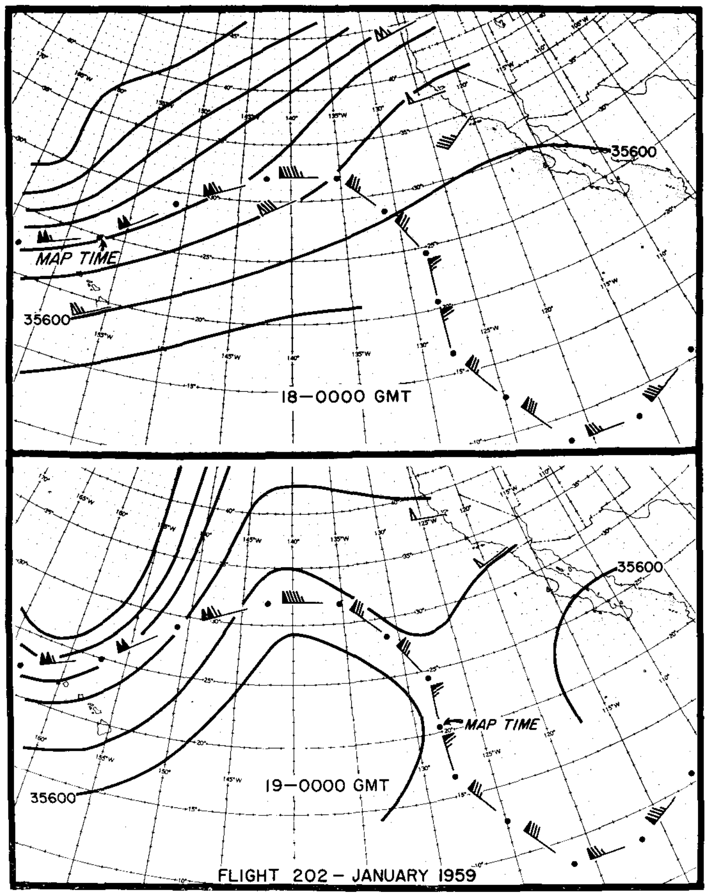

FIGURE 5.—Portion of trajectory of 250-mb. transosonde flight 202, with smoothed positions and winds (4-hour intervals) superimposed on 250-mb. NAWAC analyses 24 hours apart.

It is noted from figure 4 that the forecast trajectory almost coincides with the contour on the synoptic map closest to transosonde launch time and thus is similar to the trajectory one would obtain assuming geostrophic flow and little change in pressure pattern with time. However, rather than curving cyclonically, the actual trajectory curved anticyclonically and ended up moving in a direction exactly opposite to that of the forecast trajectory. In fairness it should be stated that there was considerable change in pressure pattern with time in this case so that the forecast trajectory error is at least partly due to error in pressure field prognosis. In particular, the trough tended to "dig" to the southwest of Baja California, more or less in agreement with the indications of the transosonde trajectory, with the result that the flow toward high pressure was not as pronounced as suggested in figure 4. One immediately wonders whether this pressure-field development was a result of the inertial instability and the consequent response of the pressure field to air-parcel trajectories such as the one illustrated by the transosonde. Further evidence in favor of such a response is presented in the next section.

5. EVIDENCE FOR THE ASSOCIATION OF INERTIAL INSTABILITY AND PRESSURE-FIELD CHANGES

In this section the question of whether inertial instability is the precursor of changes in the pressure pattern is dealt with specifically. On the one hand it appears reasonable that the large ageostrophic flows engendered by such an instability, as illustrated by the aforementioned transosonde flights, would be associated with divergence patterns which would induce changes in the pressure field. On the other hand, pressure changes at a level are the result of divergence patterns everywhere above that level, and since the criterion of inertial instability is likely to be realized only in a relatively shallow layer embracing the jet stream (say between pressure surfaces of 300 and 200 mb.), it is not obvious that the pressure field at 300 or 250 mb. *should* respond to the large ageostrophic flows at these levels associated with inertial instability. This problem of the mutual adjustment of mass and velocity fields has been treated theoretically by several authors, most recently, perhaps, by Bolin [10].

We present here two examples suggesting, but not proving, that the pressure field does respond to large ageostrophic flows associated with inertial instability at jet stream level. The top diagram of figure 5 shows the 250-mb. NAWAC analysis at the time transosonde flight 202 was located just to the north of Hawaii. On the basis of this analysis, which appears well established by the wind at Weather Ship N, one would expect the transosonde to cross the west coast of the United States within 24 hours, probably near the Oregon-California border. However, note that at map time the transosonde was located within a region of strong anticyclonic shear of the geostrophic wind. In this southerly latitude, this shear is easily sufficient to render the flow inertially unstable. The transosonde trajectory subsequent to passage through this presumed region of inertial instability is quite unexpected; far from entering the United States the transosonde curved anticyclonically and finally attained a latitude of 8°. The bottom diagram of figure 5 shows the NAWAC analysis 24 hours later, of necessity based largely on transosonde positions and winds as there were no other data available in the area of interest. A large trough is indicated in an area where none was indicated before and the existence of this trough is confirmed by later winds at Weather Ship N. There are two possibilities to be considered here. The first is that the NAWAC analysis for January 18 was incorrect and that a trough or Low already existed to the southwest of Baja California. The winds along the coast and at Weather Ship N make the presence of a trough unlikely but a cut-off Low could have been present in the region of no data. The second possibility is that a very rapid development took place, associated with the loss of geostrophic control in the region of inertial instability and the subsequent southward flow of air parcels in accordance with the transosonde trajectory. This second possibility is not too remote because, as mentioned previously, so long as the absolute vorticity is zero, it is insensitive to the field of horizontal divergence, and hence conservation of zero absolute vorticity is much more likely than conservation of a non-zero value of absolute vorticity. Thus, particularly at this low latitude, an air parcel initially embedded in a region of inertial instability could easily deviate far from its original contour channel. On flight 202, as on flight 990, there was a tendency for conservation of absolute vorticity following the transosonde as seen from the fact that for a 16-hour period, between Hawaii and the west coast, the flow was on the verge of being abnormal so that only a small anticyclonic shear would have been needed to fulfill the criterion of zero absolute vorticity.

The sparsity of data in the area under discussion in the above case does not permit a completely satisfactory analysis. Consequently, in this next example we illustrate the probable effects of inertial instability in a region of good data coverage. The three diagrams in figure 6 show NAWAC maps at 24-hour intervals during the time of passage over the United States of transosonde flight 229. The top diagram shows the transosonde embedded in a flow which would be expected to carry it north into Canada. However, at map time on the 21st the transosonde was decelerating rapidly, as shown by the speed decrease of 35 kt. during the previous 4 hours. The transosonde continued to decelerate and instead of veering northeastward in agreement with the

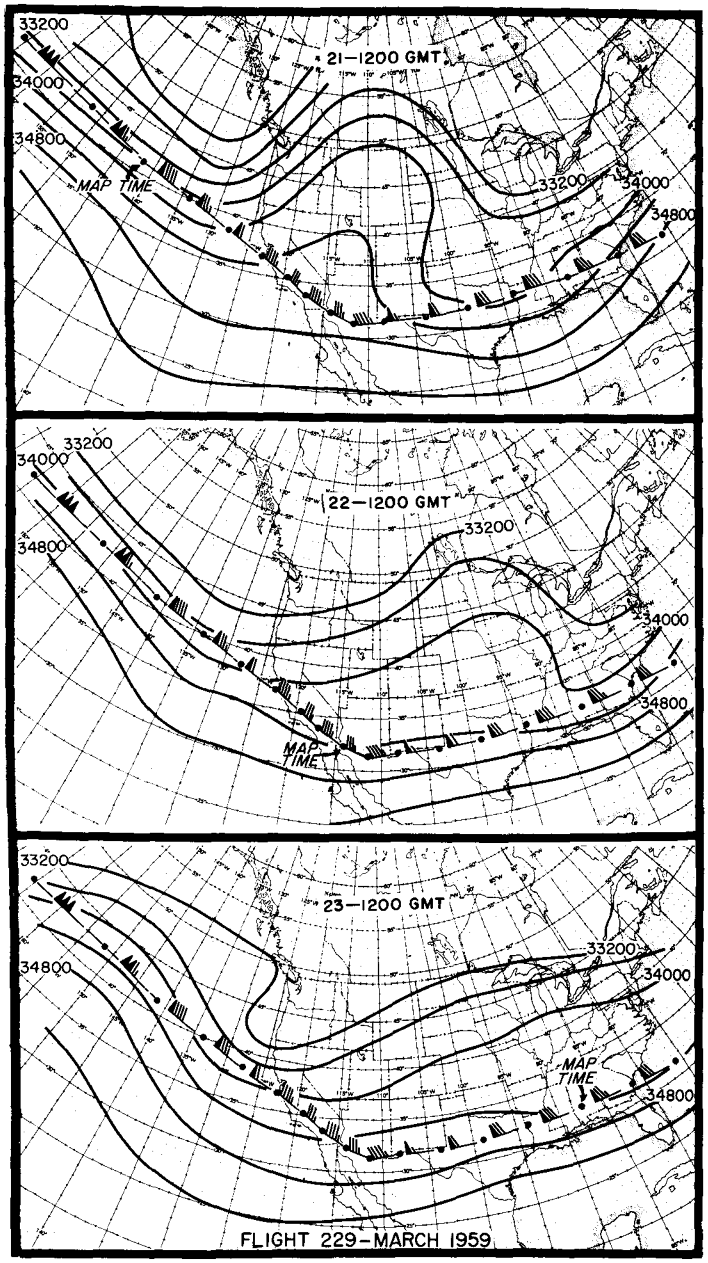

FIGURE 6.—Portion of trajectory of 250-mb. transosonde flight 229, with smoothed positions and winds (4-hour intervals) superimposed on 250-mb. NAWAC analyses 24 hours apart.

contour pattern, continued to move southeastward in a straight line and finally became embedded in the nearly zonal flow through the southern tier of States, as shown in the middle diagram. If we accept the contour analyses, we see that an area of negative absolute vorticity existed to the west of northern California where the transosonde continued to move in a straight line rather than in accordance with the contours. The three diagrams show that, at the same time that the flow (as delineated by the transosonde) cut through the base of the ridge initially over the Rocky Mountains, the ridge collapsed, as if in response to the new flow pattern set up due to the inertial instability. However, there is a problem of cause and effect here and one might also say that the collapse of the ridge enabled the flow to assume its new orientation. It is not obvious how this question of cause and effect can be answered satisfactorily except to analyze all pressure field changes subsequent to passage of transosondes through regions of inertial instability. If, in general, the pressure field does reorient in accordance with the transosonde trajectory, this represents strong evidence in favor of the flow being the cause rather than the effect of pressure field changes. If such turns out to be the case, the forecast implications are obvious. As it stands now, it is believed sufficient evidence has been presented to make worthwhile the careful consideration of regions of inertial instability and their possible influence on the forecast of flow and pressure fields.

6. CONCLUSION

Constant-level balloons represent the most obvious way, and perhaps the only way, of delineating the true importance of regions of inertial instability. While the above analysis would be more convincing and satisfying if carried out on an isentropic, rather than an isobaric, surface, presumably isobaric trajectories are sufficiently similar to isentropic trajectories so that the main features brought out in this paper are valid, namely, (1) that extremely large deviations from geostrophic flow occur in and downstream from regions of inertial instability; (2) that these deviations may well be precursors of changes in pressure pattern; (3) that trajectory cusps and small anticyclonic loops occur downstream from regions of inertial instability, with the particular configuration probably a function of the pressure-gradient magnitude in the area where the transosonde velocity becomes small. Because of its importance, further analysis is planned with regard to point (2).

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